NONSTEADY TEMPERATURE FIELD IN A THIN PLATE BETWEEN TWO TUBES INTERACTING WITH FLOWS OF HEAT CARRIERS

A. M. Makarov, L. A. Luneva, and N. I. Zalogina

UDC 536.242.01

The article states and solves the problem of a nonsteady temperature field in a thin plate between two tubes interacting with flows of heat carrier in the case of unidirectional, heterodirectional, loop, and circulating flow by the method of integral Laguerre transformation with respect to a time variable.

The analytical investigation of nonsteady processes of conjugate heat exchange in elements of power generating installations still remains one of the topical problems of modern theoretical heat engineering. The known classical methods of solving linear initial-boundary problems of conjugated heat conduction and of complex heat exchange such as: the method of Laplace integral transformation, the method of finite integral transformations, the method of boundary value problems, variational methods, etc. [1-5] yielded a number of theoretically and practically important results. The method of integral Laguerre transformation with respect to a time variable [6-8] is successful in many cases when the use of finite integral transformations is made difficult by the complexity of constructing the kernel of transformation [9, 10].

Let us consider the problem of the nonsteady temperature field in a thin plate bordered at the edges by two tubes interacting with flows of heat carriers (see Fig. 1). We assume that the thermal conductivity of the material of the plate and of the materials of the tubes along the z-axis may be neglected. This is correct if the distance between the tubes is considerably smaller than the characteristic longitudinal dimension of the problem on which the temperature field changes substantially. We assume that the walls of the tubes are thin, and that there are conditions of ideal thermal contact on the lines of the points of the plate with the tubes. We take it that the thermophysical properties of the material, the heat exchange coefficients with the flows of heat carrier and with the environment, and also the thicknesses of the plate and the tube walls do not depend on the sought temperature, on the spatial independent variables, or on the independent time variable.

To describe the processes of interaction of the flows of heat carriers with the tubes we use the model of "ideal displacement" which takes into account the effects of possible heat accumulation in an elementary volume of the medium in consequence of the finite values of specific heat capacity of the flows. For the sake of simplicity we assume that the flow rates of the heat carriers are constant in time.

In formulating the problem we assume as known the temperature distribution over the plate, the tube walls, and in the flows of heat carrier at the initial instant, and at the subsequent instants the ambient temperature and the temperature of the flows of heat carrier

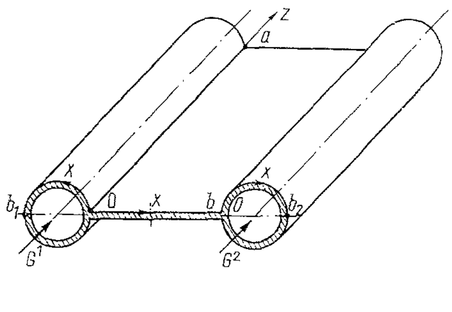

Fig. 1. Calculation diagram.

40 Years October Balashikhino Scientific and Production Association of Cryogenic Machinery. Translated fro Inzhenerno-fizicheskii Zhurnal, Vol. 61, No. 2, pp. 244-252, August, 1991. Original article submitted July 4, 1990.

"at the inlet" to the heat exchanger whose position is connected with the actual organization of movement of the heat carrier (parallel flow, counterflow, or loop flow).

We note that for the suggested method of solution a substantial constraint is the assumption that the statement of the problem is linear; assumptions that the thermophysical parameters, heat exchange and heat transfer coefficients, the thicknesses of the plate and the tube walls, and the flow rates of the heat carriers are constant, simplify the calculations, but they are not mandatory.

We write the differential equation and the initial and boundary conditions for the temperature field in the plate:

$$c\rho\delta\frac{\partial\theta}{\partial t}=\lambda\delta\frac{\partial^2\theta}{\partial x^2}-\alpha(\theta-T_{amb}), \tag{1}$$

$$\theta(0,\ x,\ z)=\theta^0(x,\ z), \tag{2}$$

$$\theta(t,\ 0,\ z)=u^1(t,\ 0,\ z), \tag{3}$$

$$\theta(t,\ b,\ z)=u^2(t,\ 0,\ z), \tag{4}$$

$$-\lambda\delta\frac{\partial\theta}{\partial x}(t,\ 0,\ z)=\lambda^1\delta^1\left(\frac{\partial u^1}{\partial x}(t,\ 0,\ z)-\frac{\partial u^1}{\partial x}(t,\ 2b_1,\ z)\right), \tag{5}$$

$$-\lambda\delta\frac{\partial\theta}{\partial x}(t,\ b,\ z)=\lambda^2\delta^2\left(-\frac{\partial u^2}{\partial x}(t,\ 0,\ z)+\frac{\partial u^2}{\partial x}(t,\ 2b_2,\ z)\right), \tag{6}$$

where $u^i(t,\ x,\ z)$, $i=\overline{1,\ 2}$ are the temperature distributions in the first and the second tube described by the following equations:

$$(c\rho\delta)^i\frac{\partial u^i}{\partial t}=(\lambda\delta)^i\frac{\partial^2 u^i}{\partial x^2}-\alpha^i(u^i-T_{amb})-\tilde{\alpha}^i(u^i-v^i), \tag{7}$$

$$u^i(0,\ x,\ z)=u^{0i}(x,\ z), \tag{8}$$

$$u^i(t,\ 0,\ z)=u^i(t,\ 2b_i,\ z). \tag{9}$$

The differential equations and the initial and boundary conditions for determining the temperatures of the heat carriers $v^i(t,\ z)$, $i=\overline{1,\ 2}$ have the form:

$$(c_p\rho F)^i\frac{\partial v^i}{\partial t}+(c_pG)^i\frac{\partial v^i}{\partial z}=\hat{\alpha}^i\int\limits_0^{2b_i}(u^i-v^i)\,dx, \tag{10}$$

$$v^i(0,\ z)=v^{0i}(z), \tag{11}$$

$$v^i(t,\ 0)=v^i_*(t). \tag{12}$$

The physical meaning of the designations used in writing the problem (1)-(12) can be easily reestablished from the form of the equations and of the initial and boundary conditions, we only note that Eqs. (10) are correct on the assumption that the temperatures in the cross sections of the tubes are constant, and the boundary conditions (12) are correct for the case $G^1 > 0$ and $G^2 > 0$ ("parallel flow").

If we assume that the temperature fields in the tubes remain symmetrical relative to the section $x = b_i$, $i=\overline{1,\ 2}$, in the course of time, then conditions (5), (6), and (9) assume the form:

$$-\lambda\delta\frac{\partial\theta}{\partial x}(t,\ 0,\ z)=2\lambda^1\delta^1\frac{\partial u^1}{\partial x}(t,\ 0,\ z), \tag{5'}$$

$$-\lambda\delta\frac{\partial\theta}{\partial x}(t,\ b,\ z)=-2\lambda^2\delta^2\frac{\partial u^2}{\partial x}(t,\ 0,\ z), \tag{6'}$$

$$\frac{\partial u^i}{\partial x}(t,\ b_i,\ z)=0,\ i=\overline{1,\ 2}. \tag{9'}$$

This entails a natural change of the right-hand side of Eq. (10). Further calculations will be carried out for the symmetric case.

To solve the problem (1)-(12) we use Laguerre transformation [6] with respect to the variable t:

$$f_n = (f;\ L_n) = \int_0^\infty f(t)\, L_n(t) \exp(-t)\, dt,\ n = 0,\ 1,\ 2,\ \ldots, \tag{13}$$

where $L_n(t)$ is a Laguerre polynomial of n-th order. For the transformation (13) the following functional properties are known:

$$f(t) = \sum_{n=0}^{\infty} f_n L_n(t), \tag{14}$$

$$\left(\frac{df}{dt};\ L_n\right) = \sum_{k=0}^{n} f_k - f(0). \tag{15}$$

It is expedient to interpret the sequence f_n as an infinite-dimensional column vector, and naturally to use the vector-matrix symbolic form of writing.

In the image space the problem (1)-(12) assumes the form:

$$\frac{\partial^2 \boldsymbol{\theta}}{\partial x^2} + A\boldsymbol{\theta} = \mathbf{f}; \tag{16}$$

$$\frac{\partial^2 \mathbf{u}^i}{\partial x^2} + M^i \mathbf{u}^i + \tilde{M}^i \mathbf{v}^i = \boldsymbol{\varphi}^i,\ i = \overline{1,\ 2}; \tag{17}$$

$$\frac{\partial \mathbf{v}^i}{\partial z} + N^i \mathbf{v}^i + \tilde{N}^i \mathbf{u}^i = \boldsymbol{\chi}^i,\ i = \overline{1,\ 2}; \tag{18}$$

$$\boldsymbol{\theta}(0,\ z) = \mathbf{u}^1(0,\ z);\ \boldsymbol{\theta}(b,\ z) = \mathbf{u}^2(0,\ z); \tag{19}$$

$$\frac{\partial \boldsymbol{\theta}}{\partial x}(0,\ z) = m_1 \frac{\partial \mathbf{u}^1}{\partial x}(0,\ z);\ \frac{\partial \boldsymbol{\theta}}{\partial x}(b,\ z) = m_2 \frac{\partial \mathbf{u}^2}{\partial x}(0,\ z); \tag{20}$$

$$\frac{\partial \mathbf{u}^i}{\partial x}(b_i,\ z) = \mathbf{0},\ i = \overline{1,\ 2}; \tag{21}$$

$$\mathbf{v}^i(0) = \mathbf{v}^i_*,\ i = \overline{1,\ 2}, \tag{22}$$

where

$$A = -\frac{c\rho}{\lambda} D - \frac{\alpha}{\lambda\delta} E;\ \mathbf{f} = -\frac{c\rho}{\lambda}\boldsymbol{\theta}^0 - \frac{\alpha}{\lambda\delta}\mathbf{T}_{\text{amb}};$$

$$M^i = -\left(\frac{c\rho}{\lambda}\right)^i D - \left(\frac{\alpha + \tilde{\alpha}}{\lambda\delta}\right)^i E;\ \tilde{M}^i = \left(\frac{\tilde{\alpha}}{\lambda\delta}\right)^i E;$$

$$\boldsymbol{\varphi}^i = -\left(\frac{c\rho}{\lambda}\right)^i \mathbf{u}^{i0} - \left(\frac{\alpha}{\lambda\delta}\right)^i \mathbf{T}_{\text{amb}};\ \boldsymbol{\chi}^i = \left(\frac{\rho F}{G}\right)\mathbf{v}^{i0};$$

$$N^i = \left(\frac{\rho F}{G}\right)^i D + \left(\frac{2\hat{\alpha} b}{c_p G}\right)^i E;\ \tilde{N}^i \mathbf{u}^i = -\left(\frac{2\hat{\alpha}}{c_p G}\right)^i \int_0^{b_i} \mathbf{u}^i dx;$$

$$m_1 = -\frac{2\lambda^1\delta^1}{\lambda\delta};\ m_2 = \frac{2\lambda^2\delta^2}{\lambda\delta};\ \boldsymbol{\theta}^0 = \text{colon}\{\theta^0,\ \theta^0,\ \theta^0,\ \ldots\};$$

$$\mathbf{u}^{i0} = \text{colon}\{u^{i0},\ u^{i0},\ u^{i0},\ \ldots\};\ \mathbf{v}^{i0} = \text{colon}\{v^{i0},\ v^{i0},\ v^{i0},\ \ldots\},\ i = \overline{1,\ 2};$$

$$\mathbf{T}_{\text{amb}} = \text{colon}\{(T_{\text{amb}})_0,\ (T_{\text{amb}})_1,\ (T_{\text{amb}})_2,\ \ldots\};$$

$$(T_{\text{amb}})_k = \int_0^\infty T_{\text{amb}} L_k \exp(-t)\, dt,\ k = 0,\ 1,\ 2,\ \ldots;$$

$$\mathbf{v}^i_* = \text{colon}\{(v^i_*)_0,\ (v^i_*)_1,\ (v^i_*)_2,\ \ldots\};\ (v^i_*)_k = \int_0^\infty v^i_* L_k \exp(-t)\, dt,\ \ k = 0,\ 1,\ 2,\ \ldots;\ \ i = \overline{1,\ 2};$$

$(E)_{ik} = \delta_{ik}$ is an identity matrix; and the matrix (D_{ki}) has elements of zero above the diagonal, and of unity on and below the diagonal.

We introduce the designations:

$$\theta_x = \frac{\partial \theta}{\partial x}; \; \mathbf{u}_x^i = \frac{\partial \mathbf{u}^i}{\partial x}, \; i = \overline{1, 2},$$

and rewrite the system of equations of second order (16), (17) in equivalent form:

$$\frac{\partial}{\partial x}\begin{pmatrix} \theta \\ \theta_x \end{pmatrix} + R\begin{pmatrix} \theta \\ \theta_x \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{f} \end{pmatrix}, \tag{23}$$

$$\frac{\partial}{\partial x}\begin{pmatrix} \mathbf{u}^i \\ \mathbf{u}_x^i \end{pmatrix} + R^i\begin{pmatrix} \mathbf{u}^i \\ \mathbf{u}_x^i \end{pmatrix} = \begin{pmatrix} 0 \\ \varphi^i - \tilde{M}^i \mathbf{v}^i \end{pmatrix}, \tag{24}$$

where the hypermatrices R and R^i are determined by the relations

$$R = \begin{pmatrix} 0 & -E \\ A & 0 \end{pmatrix}; \; R^i = \begin{pmatrix} 0 & -E \\ M^i & 0 \end{pmatrix},$$

here 0 denotes the null matrix all of whose elements are equal to zero, and 0 denotes the null vector all of whose elements are also equal to zero.

The common solutions of Eqs. (23) and (24) are conveniently written in the following form:

$$\begin{gathered} \theta = \Phi_{11}(x, 0)\,\theta(0) + \Phi_{12}(x, 0)\,\theta_x(0) + \mathbf{r}_1, \\ \theta_x = \Phi_{21}(x, 0)\,\theta(0) + \Phi_{22}(x, 0)\,\theta_x(0) + \mathbf{r}_2, \\ \mathbf{u}^i = \Phi_{11}^i(x, 0)\,\mathbf{u}^i(0) + \Phi_{12}^i(x, 0)\,\mathbf{u}_x^i(0) + \mathbf{r}_1^i + \mathrm{E}^i \mathbf{v}^i, \\ \mathbf{u}_x^i = \Phi_{21}^i(x, 0)\,\mathbf{u}^i(0) + \Phi_{22}^i(x, 0)\,\mathbf{u}_x^i(0) + \mathbf{r}_2^i + \mathrm{E}_x^i \mathbf{v}^i, \end{gathered} \tag{25}$$

where the hypermatrix is

$$\Phi(x, \xi) = \begin{pmatrix} \Phi_{11}(x, \xi) & \Phi_{12}(x, \xi) \\ \Phi_{21}(x, \xi) & \Phi_{22}(x, \xi) \end{pmatrix} = \exp[-R(x-\xi)];$$

the hypermatrix $\Phi^i(x, \xi)$ is determined analogously:

$$\mathbf{r}_1 = \int_0^x \Phi_{12}(x, \xi)\,\mathbf{f}(\xi)\,d\xi; \; \mathbf{r}_2 = \int_0^x \Phi_{22}(x, \xi)\,\mathbf{f}(\xi)\,d\xi; \; \mathbf{r}_1^i = \int_0^x \Phi_{12}^i(x, \xi)\,\varphi^i(\xi)\,d\xi; \; \mathbf{r}_2^i = \int_0^x \Phi_{22}^i(x, \xi)\,\varphi^i(\xi)\,d\xi;$$

$$\mathrm{E}^i = -\int_0^x \Phi_{12}^i(x, \xi)\,d\xi \tilde{M}^i; \; \mathrm{E}_x^i = -\int_0^x \Phi_{22}^i(x, \xi)\,d\xi \tilde{M}^i.$$

It is easy to see that the solution of the problem in the form (25) is fully determinate if the vectors $\mathbf{v}^i$, i = 1, 2 (to be determined below) are known, and the components of the block column vector are:

$$\mathbf{g} = \mathrm{colon}\,\{\theta(0)^{\mathrm{T}}, \; \theta_x(0)^{\mathrm{T}}, \; \mathbf{u}^1(0)^{\mathrm{T}}, \; \mathbf{u}_x^1(0)^{\mathrm{T}}, \; \mathbf{u}^2(0)^{\mathrm{T}}, \; \mathbf{u}_x^2(0)^{\mathrm{T}}\},$$

where the superscript T denotes the oepration of transposition. Requiring that the boundary conditions (19)-(21) be fulfilled, we arrive at the following system of equations for determining the sought components of the vector g:

$$K\mathbf{g} = \mathbf{s},$$

where the nonzero elements of the hypermatrix K are determined by the relations

$$\begin{gathered} K_{11} = -K_{13} = K_{22} = -K_{35} = E, \; K_{24} = -m_1 E, \; K_{46} = -m_2 E, \\ K_{31} = \Phi_{11}(b, 0), \; K_{32} = \Phi_{12}(b, 0), \; K_{41} = \Phi_{21}(b, 0), \; K_{42} = \Phi_{22}(b, 0), \\ K_{53} = \Phi_{21}^1(b_1, 0), \; K_{54} = \Phi_{22}^1(b_1, 0), \; K_{65} = \Phi_{21}^2(b_2, 0), \; K_{66} = \Phi_{22}^2(b_2, 0). \end{gathered}$$

The components of the block column vector of the right-hand sides are conveniently written in the form

$$\begin{gathered} \mathbf{s} = \mathbf{s}_0 + \lambda, \; \mathbf{s}_0 = -\mathrm{colon}\,\{0^{\mathrm{T}}, \; 0^{\mathrm{T}}, \; \mathbf{r}_1(b)^{\mathrm{T}}, \; \mathbf{r}_2(b)^{\mathrm{T}}, \; \mathbf{r}_2^1(b_1), \; \mathbf{r}_2^2(b_2)^{\mathrm{T}}\}, \\ \lambda = -\mathrm{colon}\,\{0^{\mathrm{T}}, \; 0^{\mathrm{T}}, \; 0^{\mathrm{T}}, \; 0^{\mathrm{T}}, \; (\mathrm{E}_x^1(b_1)\,\mathbf{v}^1)^{\mathrm{T}}, \; (\mathrm{E}_x^2(b_2)\,\mathbf{v}^2)^{\mathrm{T}}\}. \end{gathered}$$

Assume that $W = K^{-1}$, then the components of the vector $\mathbf{g}$ are determined by the relation

$$\mathbf{g} = W\mathbf{s}_0 + W\boldsymbol{\lambda},$$

from which follows that all the components of the vector $\mathbf{g}$ depend linearly on $\mathbf{v}^1$ and $\mathbf{v}^2$.

Let us revert to Eq. (18) with the initial conditions (22). We note that the expression $\tilde{N}^i \mathbf{u}^i$ consists of two terms, one of which is some function of the z-coordinate, and the other is proportional to the sought function $\mathbf{v}^i$, where the structure of the expression $\tilde{N}^i \mathbf{u}^i$ is such that Eq. (18) is a system of "interconnected" equations. Indeed, the sought dependences $\mathbf{u}^i(x)$ can be written in the form

$$\mathbf{u}^i(x) = \boldsymbol{\rho}^i(x) + A^i_*(x)\,\mathbf{v}^1 + B^i_*(x)\,\mathbf{v}^2, \tag{26}$$

where

$$\boldsymbol{\rho}^i = \Phi^i_{11}(x,\ 0)(W\mathbf{s}_0)_{i+2} + \Phi^i_{12}(x,\ 0)(W\mathbf{s}_0)_{i+3} + \mathbf{r}^i_1(x);$$

$$A^i_* = (\Phi^i_{11}(x,\ 0)\,W_{i+2,5} + \Phi^i_{12}(x,\ 0)\,W_{i+3,5})\,\mathrm{E}^1_x(b_1) + (2-i)\,\mathrm{E}^1(x);$$

$$B^i_* = (\Phi^i_{11}(x,\ 0)\,W_{i+2,6} + \Phi^i_{12}(x,\ 0)\,W_{i+3,6})\,\mathrm{E}^2_x(b_2) + (i-1)\,\mathrm{E}^2(x);$$

here expression W_{ij} is a square matrix element (ij) of the hypermatrix W whose dimensionality is 6 × 6; expression $(W\mathbf{s}_0)_j$ is the element j of the block column vector (i.e., the column vector of corresponding dimensionality).

After substituting relation (26) into the differential equation (18) we obtain

$$\frac{\partial \mathbf{v}}{\partial z} + G\mathbf{v} = \mathbf{h}, \tag{27}$$

where

$$\mathbf{v} = \begin{pmatrix} \mathbf{v}^1 \\ \mathbf{v}^2 \end{pmatrix};\ G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix};\ \mathbf{h} = \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \end{pmatrix};\ \mathbf{h}^i = \boldsymbol{\chi}^i + \left(\frac{2\hat{\alpha}}{c_pG}\right)^i \int_0^{b_i} \boldsymbol{\rho}^i(x)\,dx, \quad i = \overline{1,\ 2};\ G_{11} = N^1 - \left(\frac{2\hat{\alpha}}{c_pG}\right)^1 \int_0^{b_1} A^1_*(x)\,dx;$$

$$G_{12} = -\left(\frac{2\hat{\alpha}}{c_pG}\right)^1 \int_0^{b_1} B^1_*(x)\,dx;\quad G_{21} = -\left(\frac{2\hat{\alpha}}{c_pG}\right)^2 \int_0^{b_2} A^2_*(x)\,dx;\ G_{22} = N^2 - \left(\frac{2\hat{\alpha}}{c_pG}\right)^2 \int_0^{b_2} B^2_*(x)\,dx.$$

The solution of Eq. (27) is known:

$$\mathbf{v} = \overset{0}{\Phi}(z,\ 0)\,\mathbf{v}(0) + \int_0^z \overset{0}{\Phi}(z,\ \xi)\,\mathbf{h}(\xi)\,d\xi, \tag{28}$$

where

$$\overset{0}{\Phi}(x,\ \xi) = \begin{pmatrix} \overset{0}{\Phi}_{11} & \overset{0}{\Phi}_{12} \\ \overset{0}{\Phi}_{21} & \overset{0}{\Phi}_{22} \end{pmatrix} = \exp\left[-G(z-\xi)\right].$$

Requiring that the initial condition (22) be fulfilled, we obtain

$$\mathbf{v}(0) = \mathbf{v}_* = \begin{pmatrix} \mathbf{v}^1_* \\ \mathbf{v}^2_* \end{pmatrix}, \tag{29}$$

which concludes the construction of the solution of the problem under consideration as a whole since in the image space all the sought functions have been determined, and the inverse transformation, with the respective assumptions fulfilled, is carried out by rule (14).

It is not difficult to examine the case of flows of heat carriers moving in "counterflow." For the sake of determinacy let the first flow be in the direction of the z-axis, and the second flow in the opposite direction. The length of the heat exchanger along the z-axis is equal to a. In that case $G^2 < 0$, and the boundary conditions (12) for the system of Eqs. (1) are replaced by the following ones:

$$v^1(t,\ 0) = v^1_*(t),\ v^2(t,\ a) = v^2_*(t). \tag{30}$$

All the results obtained above including relation (28) remain valid, only relation (29) requires some amendment. In accordance with conditions (30) we have:

$$\mathbf{v}^1(0) = \mathbf{v}^1_*$$

and then

$$\mathbf{v}^2(0) = (\overset{0}{\Phi}_{22}(a,\ 0))^{-1}(\mathbf{v}^2_* - \mathbf{y}^2(a) - \overset{0}{\Phi}_{21}(a,\ 0)\,\mathbf{v}^1_*),$$

where

$$\mathbf{y}(z) = \begin{pmatrix} \mathbf{y}^1 \\ \mathbf{y}^2 \end{pmatrix} = \int_0^z \overset{0}{\Phi}(z,\ \xi)\,\mathbf{h}(\xi)\,d\xi.$$

Thus the block column vector $\mathbf{v}(0)$ in relation (28) has been fully determined, the solution is concluded.

Let us consider the case of loop motion of the flow of heat carrier in a heat exchanger. As before, $G^1 > 0$, and $G^2 < 0$. We note that loop motion of flows with decreased or increased flow rate in the section $z = a$, may be considered, below therefore $|G^2| \lesseqgtr G^1$. Moreover, increased flow rate in the "reverse" flow can lead to a change of temperature of the flow in the section $z = a$. An analogous effect is attained by localized supply (removal) of thermal power to (from) the flow. In the general case of loop motion the boundary conditions for the system or equations (10) have the form

$$v^1(0,\ t) = v^1_*(t),\ v^2(a,\ t) = v^1(a,\ t) + q(t). \tag{31}$$

In accordance with condition (31) we have

$$\mathbf{v}^1(0) = \mathbf{v}^1_*,\ \ \mathbf{v}^2(0) = (\overset{0}{\Phi}_{12}(a,\ 0) - \overset{0}{\Phi}_{22}(a,\ 0))^{-1}[\mathbf{q} + \mathbf{y}^2(a) - \mathbf{y}^1(a)\ + (\overset{0}{\Phi}_{21}(a,\ 0) - \overset{0}{\Phi}_{11}(a,\ 0))\,\mathbf{v}^1_*],$$

which fully solves the stated problem. attained

If in the heat exchanger under consideration circulation of the heat carrier is arranged in a closed circuit ($G^1 > 0$, $G^2 < 0$, $|G^2| \lesseqgtr G^1$) with two active sections affecting the flow of heat carrier $z = 0$ and $z = a$, then the system of boundary conditions for Eqs. (10) assumes the form

$$v^1(0,\ t) = v^2(0,\ t) + q^0(t),\ v^2(a,\ t) = v^1(a,\ t) - q^a(t). \tag{32}$$

With a view to expression (28) relations (32) in the image space assume the form

$$\mathbf{v}^1(0) = \mathbf{v}^2(0) + \mathbf{q}^0,\ \overset{0}{\Phi}_{11}(a,\ 0)\,\mathbf{v}^1(0) + \overset{0}{\Phi}_{12}(a,\ 0)\,\mathbf{v}^2(0) + \mathbf{y}^1(a) =$$

$$= \overset{0}{\Phi}_{21}(a,\ 0)\,\mathbf{v}^1(0) + \overset{0}{\Phi}_{22}(a,\ 0)\,\mathbf{v}^2(0) + \mathbf{y}^2(a) - \mathbf{q}^a.$$

We substitute the first of the obtained relations into the second one and solve it for $\mathbf{v}^2(0)$:

$$\mathbf{v}^2(0) = (\overset{0}{\Phi}_{11}(a,\ 0) + \overset{0}{\Phi}_{12}(a,\ 0) - \overset{0}{\Phi}_{21}(a,\ 0) - \overset{0}{\Phi}_{22}(a,\ 0))^{-1} \times$$

$$\times\ [\mathbf{y}^2(a) - \mathbf{y}^1(a) - \mathbf{q}^a + (\overset{0}{\Phi}_{21}(a,\ 0) - \overset{0}{\Phi}_{11}(a,\ 0))\,\mathbf{q}^0].$$

Thus, in this case, too, the solution of the problem can be concluded.

To illustrate the possibilities of the method we calculate the temperature distribution over the plate, the tube walls, and in the specified section of the flow of heat carriers for "parallel flow" on the assumption that: the initial distributions of temperatures over the heat exchanger (including the temperature of the flows) are uniform and taken as the reference point, flows of heat carrier with dimensionless temperature equal to unity are supplied to the inlet of the heat exchanger; there is no heat exchange with the environment; the dimensionless length of the tubes is $a = 1$, the dimensionless width of the plate is $b = 0.1$, the dimensionless size of the half-perimeter of the cross section of the tube is $b_1 = b_2 = 0.1$, the factors of thermal connection of the tube walls with the plate are $m_1 = -m_2 = -2$; the dimensionless flow velocities were adopted equal to 10, the dimensionless heat

TABLE 1. Results of the Calculation

Temperature	t = 0,1	0,5	1,0
	$z = 0{,}2;\ x = 0{,}06$		
θ	0,0917	0,172	0,272
$u^{1,2}$	0,0925	0,173	0,273
$v^{1,2}$	0,937	0,945	0,954
	$z = 0{,}6;\ x = 0{,}06$		
θ	0,0739	0,151	0,248
$u^{1,2}$	0,0746	0,152	0,249
$v^{1,2}$	0,819	0,842	0,869

exchange coefficients between the tube and the flow of heat carrier equal to 0.5; the dimensionless heat exchange coefficients between the flow of heat carrier and the tube wall equal to 0.058, the dimensionless values of the "thermal diffusivity" equal to unity. In the solution, two terms of the expansion were retained (for $L_0 = 1$ and $L_1 = 1 - t$), in the calculation of the matrix exponent by the recurrent method 8 terms of the expansion were used. It can be seen from Table 1 that even with the adopted approximation the solutioin is in full agreement with the physical meaning of the problem. In fact, the heating of the plate lags somewhat behind the heating of the material of the tube wall, sections further from the inlet to the heat exchanger are heated more slowly than nearer ones. The table does not show the symmetry of the results of the calculation of the temperature in the sections x = const symmetrically arranged about the center of the plate, nor does it show that in the section far from the plate the tube wall is heated more intensely than in the section near the plate.

Thus the problems under consideration, dealing with the nonsteady temperature field in a thin plate between two tubes interacting with the flows of heat carriers, may be regarded as concluded. There are no difficulties in principle in extending the method of integral Laguerre transformations to the case of time-dependent variables of the flow rates of heat carriers, variable parameters of heat exchange with respect to x and t, and to cases of nonideal thermal contact between the tube walls and the edges of a plate.

NOTATION

θ, u, v) temperature; t) time; x, z) coordinates; δ, δ_1, δ_2) thicknesses of the plate wall and the tube; b, b_1, b_2) plate width and half-perimeters of the cross section of the tubes; F) cross-sectional area of the tube; G) flow rate; c, c_p) heat capacity of the material and of the flow, respectively; λ) thermal conductivity of the material; α, $\tilde{\alpha}$, $\hat{\alpha}$) heat transfer coefficients; T_{amb}) ambient temperature; q) reduced thermal effect.

LITERATURE CITED

1. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press, New York (1959).
2. É. M. Kartashov, Analytical Methods in the Theory of Thermal Conductivity of Solids [in Russian], Moscow (1985).
3. V. A. Ditkin and A. P. Prudnikov, Integral Transformations and Operational Calculus [in Russian], Moscow (1961).
4. A. S. Galitsyn and A. N. Zhukovskii, Integral Transformations and Special Functions in Problems of Thermal Conductivity [in Russian], Kiev (1976).
5. V. S. Zarubin, Engineering Methods of Solving Problems of Thermal Conductivity [in Russian], Moscow (1983).
6. I. M. Vinogradov (ed.), Mathematical Encyclopedia [in Russian], Moscow (1982).
7. A. M. Makarov and V. P. Afonina, Izv. Akad Nauk SSSR, Énerg. Transp., No. 3, 69-77 (1989).
8. A. M. Makarov and T. A. Agupova, Inzh.-fiz. Zh., 55, No. 4, 670 (1988). Deposited at VINITI May 3, 1988, reg. No. 3361-V88.
9. A. M. Makarov and V. R. Romanovskii, Fiz. Khim. Obrab. Mater., No. 1, 44-50 (1981).
10. A. M. Makarov and V. R. Romanovskii, Fiz. Khim. Obrab. Mater., No. 1, 16-21 (1980).